JOURNAL OF MATERIALS SCIENCE **40** (2005) 87–95

Mechanism and behavior of bitumen strength reinforcement using fibers

JIAN-SHIUH CHEN*, KUEI-YI LIN
Department of Civil Engineering, National Cheng Kung University, Tainan 701, Taiwan
E-mail: jishchen@mail.ncku.edu.tw

This paper investigates the reinforcement mechanism of bitumen mixed with fibers. Fibers including cellulose, rock wool and polyester types were added to bitumen. The viscosity, toughness and tenacity, microscopy and rheological tests were conducted to characterize the engineering properties of bitumen-fiber mastics. Test results indicate that the reinforcing effect increases with increasing fibers up to a critical fraction. With higher mixing temperatures, there is a higher viscosity ratio of mastic to bitumen. The tensile strength of bitumen-fiber mastics also increases with increasing fiber concentrations because the fibers carry parts of tensile loads. With the increasing tensile strength, it is implied that there is a good adhesion between bitumen and fibers. Scanning electron micrographs show that fibers reinforce bitumen through a three dimensional structure. However, there is a critical fiber fraction when fibers start to interact with each other, resulting in lower toughness. The optimum fiber content is dependent on fiber type, length and diameter.

1. Introduction

Adding fibers into bitumen, also referred to as asphalt binder, is a relatively new concept in asphalt pavement technology, although many earlier applications with fibers have been reported [1, 2]. Originally, the potential of fiber used as a construction material in cement concretes and lightweight structures was recognized more than 50 years ago. Recent surveys of highway agencies that use stone matrix asphalt and porous asphalt indicate that in Europe, USA and other countries almost every project uses some form of fibers [3–11]. Three types of fibers, i.e., organic, mineral and polyester fibers, are mixed with bitumen to stabilize the mastic and reduce binder drainage, and this mixture is called bitumen-fiber mastics. Fibers should reinforce the binder phase, but different kinds of fibers are added in different amounts, so their use should be optimized.

In bitumen-fiber mastics, bitumen can be called the matrix material, the characteristics of which are changed by using fibers in the matrix as stabilizing additives. Fibers are traditionally added to prevent the binder from draining out when the asphalt mixture is hot. The mastic that is composed of fibers and bitumen can be considered to be the medium that actually bonds the aggregate together, thus becoming an essential part of hot-mix asphalt concrete. The mechanism of fibers affecting bitumen is complex, and its impact upon pavement performance is profound. As fibers are mixed with bitumen, they increase binder stiffness, which may cause brittleness in the asphalt mixture. Too much stiffening could lead to the development of pavement distress such as disintegration and fracture under the influence of climate and traffic loading. The understanding of the properties of bitumen-fiber mastics helps better control the performance of asphalt pavements. Bitumen-fiber mastics are, however, poorly characterized scientifically.

The high cost of fibers compared to bitumen makes the commercial use of fibers only attractive for road construction if the amount of fibers needed to significantly improve pavement performance is relatively small. Thus, it is imperative to determine the optimal fiber content that improves bitumen properties to a satisfactory level at a minimum cost. Therefore, the objectives of this study are (1) to characterize the behavior of bitumen-fiber mastics, (2) to evaluate the effect of fibers on bitumen, and (3) to decide the optimum content of fibers mixed with bitumen.

2. Materials

2.1. Bitumen

The bitumen employed in this experimental study was grade AC-20 according to ASTM D 3381. This material, supplied by the China Petroleum Cooperation, is the usual bitumen grade used for asphalt pavements in Taiwan.

2.2. Fibers

Four fibers including two loose polyester fibers, a loose wool-like mineral fiber, and a palletized organic fiber

*Author to whom all correspondence should be addressed.

TABLE I Properties of fibers

Fiber	Source	Specific gravity	Length (μm)	Diameter (μm)	Tensile strength (MPa)	Specific surface area (m^2/g)
Polyester-l	Polypropylene	1.18	12,000	90	330	0.07
Polyester-s	Polypropylene	1.18	6,000	90	330	0.08
Mineral	Basalt	2.75	6,000	6	2	0.10
Organic	Cellulose	1.15	1,100	75	N.A.	1.50

were used as a stabilizer to mix with bitumen. Fibers employed in hot mix asphalt mixtures are primarily used to prevent binder draindown particularly for stone matrix asphalt and porous asphalt during the mixing, transportation and compaction processes. Table I lists the basic properties of these four fibers. The polyester fibers are made of polypropylene spun into two different lengths 12,000 and 6,000 μm, which are represented by polyester-l and polyester-s, respectively. Polypropylene traditionally used for apparel and home furnishings has been recently applied to road construction because of high tensile strength. Polyester fibers have a melting point of 280°C, thus remaining intact during high mixing temperatures. Mineral fibers comprise spun fibers produced by extruding molten basal mineral through fine spinnerets. Mineral fibers possess the highest specific gravity among four fibers. Organic fibers comprise finely milled cellulose contents derived directly from wood. The cellulose fiber is gray with the average length of 1,100 μm, which is the shortest. Both mineral and organic fibers have been used for asphalt pavements since early 1980s [3, 4, 6].

3. Preparation of bitumen-fiber mastics

Fibers were first put into a 165°C oven for 24 h to ensure moisture-free fiber surfaces, and bitumen stored in a one-quart can was preheated in the can for two hours in a 165°C oven to make bitumen liquid and ready to mix. An experimental protocol was developed to obtain homogeneous bitumen-fiber mastics. The mixer, a model of Eyel 4 produced by the Kikakikae Company in Tokyo, applies a constant mixing speed to ensure no voids are created in the mixtures. In order to investigate the effects of fibers on bitumen, a full range of concentrations covering 0, 0.1, 0.2, 0.3, 0.4 and 0.5 percent by weight of the asphalt mixture were tested. The concentration of 0% represents pure bitumen, and was used as the control mix.

In preparation, 600 g of the heated bitumen was poured into a 2000-ml spherical flask, which was then placed in a heating mantle. To avoid the adverse effects of excessive heat, the temperature was carefully monitored throughout mixing using two thermocouple probes. The first probe, which was installed between the beaker and heating mantle, controlled the power input. The second one directly measured the temperature of the binder inside the beaker. Upon reaching 165°C, a pre-weighed amount of fibers was slowly added to the bitumen, while the mechanical stirring was continued at 500 rpm to prevent the fibers from possible agglomeration. Mixing was then continued at 165°C for two hours to produce homogeneous bitumen-fiber mastics. After completion, the bitumen-fiber mastic was removed from the flask and divided into small containers. The blend was cooled to room temperature, sealed with aluminum foil and stored for further testing.

4. Test methods

4.1. Scanning electron microscopy (SEM)

Scanning electron microscopy used in this study permits the direct observation of bitumen-fiber mastics. A preparation method was used to leak out the oil phase by solvent without disturbing the binder, and deoiled samples on filter paper were metallized and observed with a Htachi Model S-2500 SEM. The acceleration tension of the electron beam was 5 kV. Hence, fissure changes in bitumen-fiber mastics were visible and the main structure was observed. This technique allows the dispersed fiber phase to be seen as bright while the continuous asphaltene-rich phase remains dark. Researchers find electron microscopy preferable because it provides a clear view of a material in its state [12, 13].

4.2. Viscosity

The viscosity at 60°C was used for grading bitumen-fiber mastics by their consistency. The time was measured for a fixed volume of the mastics to be drawn up through a capillary tube by means of vacuum, under closely controlled vacuum conditions (300 ± 0.5 mm Hg vacuum) according to ASTM D 2170. The viscosity in poise was calculated by multiplying the flow time in seconds by the viscometer calibration factor.

4.3. Softening point

Softening points were used to determine the temperature at which a phase change occurs in bitumen-fiber mastics. A steel ball weighing 3.5 g was placed on a disk of mastic sample contained in a vertically supported, 20-cm diameter metal ring according to ASTM D 36. The assembly was heated in an ethylene glycol bath at 5°C/min. The softening point was taken as the temperature at which the sample became soft enough to allow the ball, enveloped in the sample material, to fall a distance of 25.4 mm. It is also known as the ring and ball softening temperature ($T_{\mathrm{r+b}}$).

4.4. Penetration test

The penetration test was to measure the consistence of bitumen mixed with different fibers. In testing, a container of bitumen-fiber mastics stored at 25°C was penetrated by a needle weighted to 100 g according to

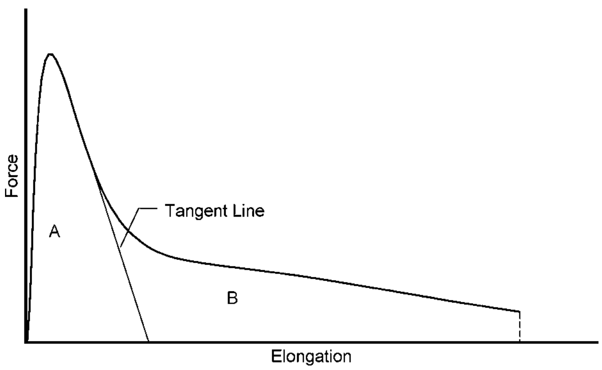


Figure 1 Typical curve from toughness and tenacity test.

ASTM D5. The distance, in units of 0.1 mm, is the penetration measurement.

4.5. Toughness and tenacity test

The toughness and tenacity test was used to monitor the tensile strength properties of fiber-reinforced bitumen. Samples were imbedded in a container with a hemispherical tension head and the sample was allowed to cool to 25°C. The head was then pulled at a rate of 50 cm/min to produce a load-deformation curve according to ASTM D 5801. A typical tensile strength versus elongation curve is shown in Fig. 1. The tenacity is defined as the area of B and the toughness is the total area under the curve, i.e., A + B.

4.6. Dynamic shear rheometer (DSR)

The rheological properties of bitumen-fiber mastics were measured by a dynamic shear rheometer (DSR) over a broad range of temperatures. The DSR was an AR-500 model manufactured by the Carri-Med Corporation. All tests were performed in the linear viscoelastic range. For tests at 40°C and higher, a 1-mm gap and a 25-mm diameter plate were used. For tests below 40°C, a 2-mm gap and an 8-mm diameter plate were used. Approximately 1 g of binder was applied to the bottom plate, covering the entire surface, and the plate was mounted in the rheometer. After heating to the test temperature of the binder, the top plate was brought into contact with the sample and the sample was trimmed. An actuator then applied a sinusoidal strain. Viscoelastic properties at different temperatures and frequencies were obtained. A specific strain level was determined at each testing temperature for each sample running a strain sweep at 100 rad/s prior to any frequency sweep. The strain was kept low enough so that all tests were performed within the linear viscoelastic range. The actual strain and torque were measured and input to a computer for calculating various viscoelastic parameters, including complex modulus (G^*) and phase angle (δ).

5. Results and discussion

5.1. Scanning electron micrograph

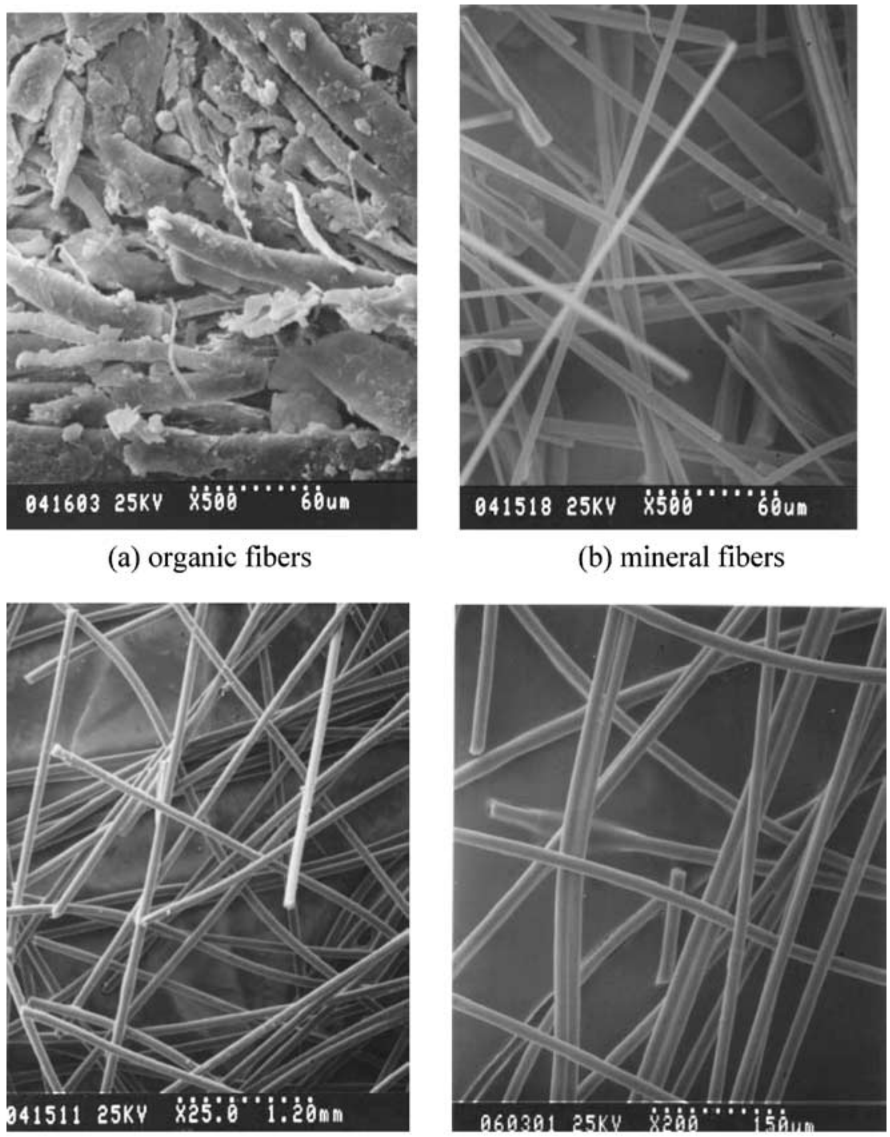
A scanning electron micrograph of four fibers is shown in Fig. 2. Organic fibers are of the ribbon type, porous and with relatively flat cross section. Some organic filaments have been torn apart, thus increasing their surface area, and their 1.5 m^2/g surface area of organic fibers is more than ten-fold greater than one of mineral and polyester fibers. The surface areas for mineral and polyester fibers are only 0.1 and 0.07 m^2/g, as listed in Table I. These surface characteristics explain the efficiency of cellulose fibers to bind more bitumen.

Note that the combination of high bitumen content and low fiber concentration may result in the binder draining by gravity from the asphalt mixture. Fig. 2 also shows that these fibers can form a three directional network in bitumen, which retains bitumen when mixed at high temperatures. This three dimensional structure of fibers could assist in the formation of a thicker coating of mastics without draining down. The microstructure of mineral and polyester fibers differs from that of cellulose fibers since the cross-section of mineral and polyester fibers is quite round, with a smooth surface and a smaller surface area. The mineral fibers are more rigid than the flexible polyester fibers, and tend to be aligned with no entanglement.

5.2. Viscosity

The viscosity increases with increasing fiber contents, as shown in Fig. 3. The viscosity ratio of organic mastics to bitumen is shown in Fig. 4 as a function of temperature. There is a limited increase in viscosity at a content of 0.1% because the organic fibers act only as a dispersing material. At 0.3% fiber concentration, the viscosity increases by five to eight times because the organic fibers begin to form a localized network structure. When the fiber content is close to 0.4%, the local networks gradually interact to initiate a continuous network throughout bitumen. The formation results in an increase in the viscosity with the increasing fiber content. This network acts as a support structure, reinforcing the bitumen and resisting deformation. Fig. 4 also shows that the viscosity ratio increases with increasing temperatures, especially at higher concentrations. This increase implies that stiffening effects are more significant at higher temperatures than at lower temperatures, which is beneficial to bitumen strength. In other words, the higher viscosity ratio could lead to reduce draindown and rutting of an asphalt mixture. The increase in the viscosity ratio of mastics primarily results from fiber reinforcement since the viscosity of bitumen AC-20 behaves like water at high temperatures. Stiff mastic and asphalt mixtures resulting from adding fibers have been observed in the laboratory as well as in the field. The addition of fibers into asphalt mixtures enhances the pavement resistance to rutting at high temperatures. Special attention should, however, be paid when excessive fibers are used. The overcrowding fibers may result in voids during the mixing process and lead to poor pavement performance.

The viscosity ratio of four fibers is shown in Fig. 5. The mineral fiber at 0.4% concentration shows a significant increase in the viscosity ratio. Thus, concentrations at 0.3% organic fibers and 0.4% mineral fibers are found to be the desirable content. The corresponding



(a) organic fibers (b) mineral fibers

(c) polyesyer-l (d) polyester-s

Figure 2 Scanning electron micrographs of four fibers.

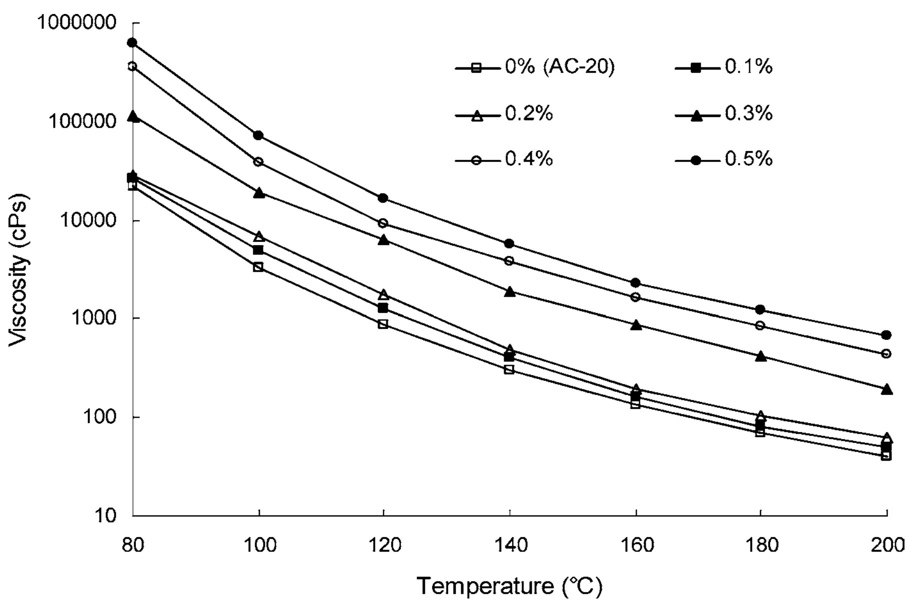


Figure 3 Viscosity of bitumen mixing with organic fibers.

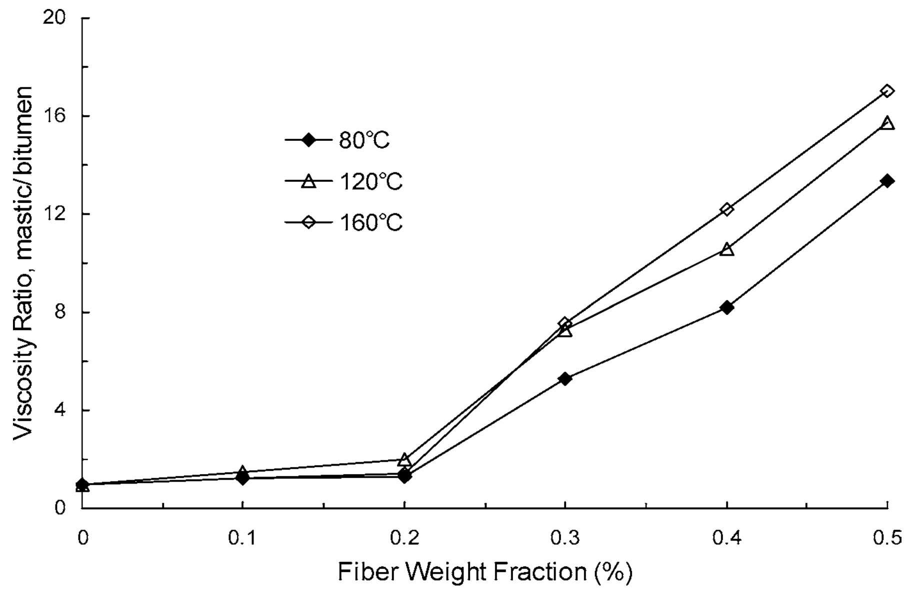


Figure 4 Viscosity ratio of organic mastic to bitumen as a function of temperature.

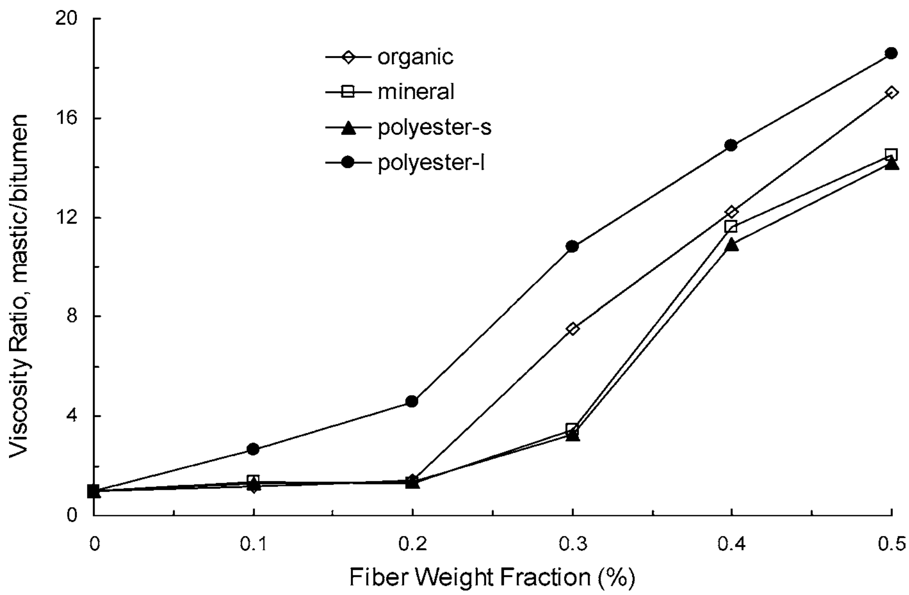


Figure 5 Viscosity ratio of mastic to bitumen for four fibers at 160°C.

viscosity ratio for the desirable content is between 8 and 12, indicating that the stiffening effect of 10 times should be adequate to reinforce bitumen. According to this reinforcement criterion, the desirable content of polyester fibers seems to be at 0.4 and 0.3% for short and long fibers, respectively. Note that the reinforcement effect is one of factors that should be considered for the optimum content of fibers. Other factors including cost, fiber-fiber interaction and construction need to be taken into account. Mixing more fibers than the optimum concentration is not economical, because too much reinforcement could lead to brittle mastics, thus deteriorating pavement performance. Adding mineral fillers to bitumen also increases the viscosity of the resulting mastic [22]. Note that short polyester fibers give less increase in viscosity than long ones. Fibers should be long enough to achieve the expected viscosity increase after being incorporated into the bitumen. Special attention should, however, be paid when long fibers may significantly increase the viscosity ratio and cause mixing problems.

5.3. Softening point and penetration

The ring-and-ball softening point temperature (T_{r+b}) is another important performance criterion for binders. The T_{r+b} value increases rapidly with a 0.2% addition of polyester fibers, as illustrated in Fig. 6. After this point the elastomeric phase in polypropylene becomes continuous, thus contributing to the steady increase in the T_{r+b}. The magnitude of the achievable increase in T_{r+b} is a function of fiber length and concentration. The maximum temperature measured in pavements is approximate 60°C. In order to combat the effect of hot summer temperatures, the fiber fraction needs to be at least 0.1, 0.3 and 0.3% by mixture weight for polyester, organic and mineral fibers, respectively. The fiber content determined by T_{r+b} serves as the minimum value

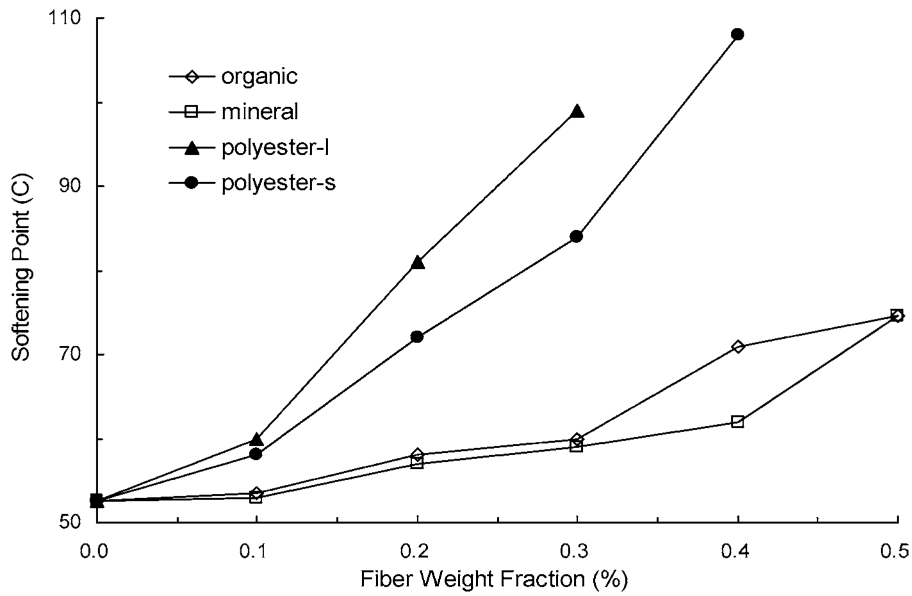


Figure 6 Softening point of bitumen mixing with four fibers.

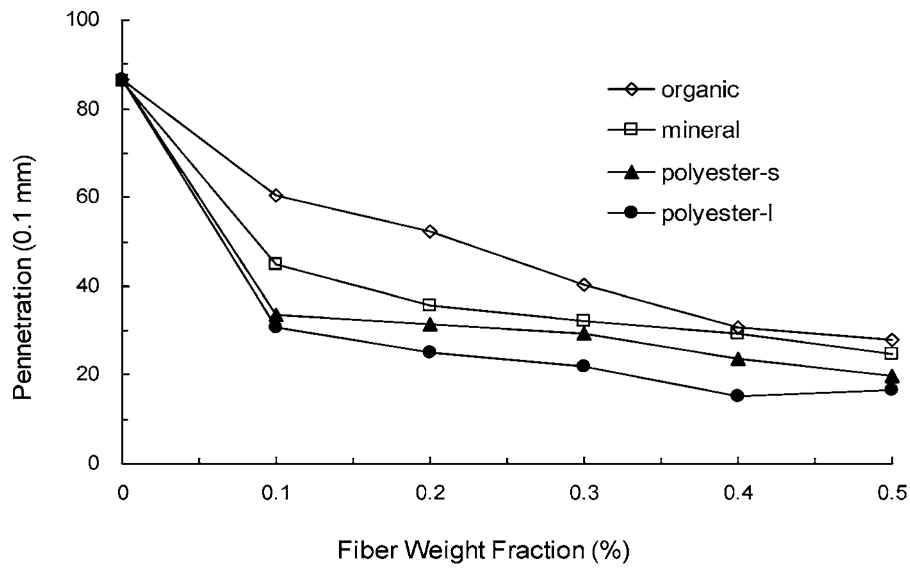


Figure 7 Penetration of bitumen mixing with four fibers.

that should be satisfied for the summer climate of Taiwan.

Polyester fibers tend to have higher softening point temperatures than organic and mineral fibers due to the fiber entanglement. Similar results are also observed by the penetration test as shown in Fig. 7, which clearly demonstrates the entanglement effect of polyester fibers. Note that polyester-s and mineral fibers have the same length, but completely different softening points and penetrations. This is because mineral fibers are rigid, so they cannot be flexibly entangled with each other. Similar changes in properties can be obtained by adding inert fillers to bitumen [23].

5.4. Tensile strength and toughness

Fig. 8 shows the variations of the tensile strength as a function of fiber fraction for bitumen-mineral fiber mastics at 25°C. The addition of mineral fibers effectively enhances the ultimate strength. Researchers have analyzed fiber-filled composites under various conditions [14–16]. Two general limits can be summarized for the tensile strength of fiber-filled composites, as shown in Fig. 9. The upper-bound response represents strong adhesion between matrix and fiber, while the lower-bound response indicates weak or no adhesion between these two phases. In this study, the addition of fibers causes an increase in tensile strength. The increase in the tensile strength with increasing amounts of fibers implies that there is good adhesion between bitumen and fibers. With this good adhesion, bitumen binders are able to hold fibers together during loading. As a result, the tensile strength of the overall system increases.

If there is good adhesion in the fiber-filled composite, the fibers carry some of the tensile loads [16, 17]. When bitumen-fiber mastics are tested under tension, the stress is transmitted from the matrix to the fiber. Part of the tensile stress can be carried by the fiber. More fibers added can share more tensile stresses with the bitumen,

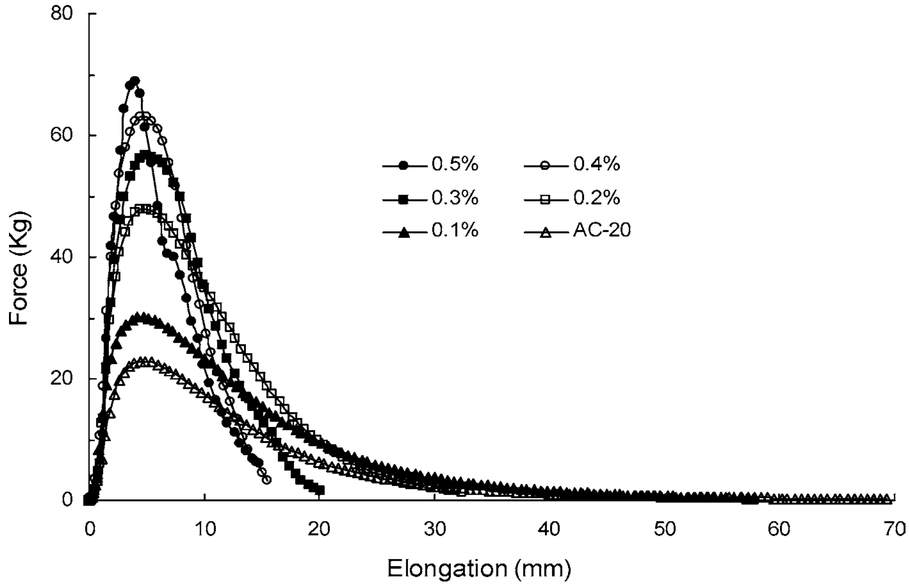


Figure 8 Tensile force versus elongation for bitumen mixing with mineral fibers at different concentrations.

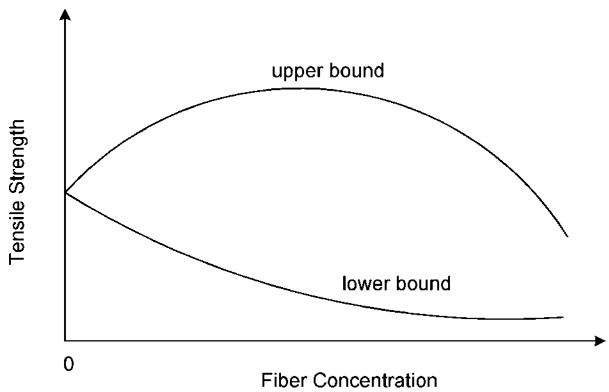


Figure 9 Effects of fibers on tensile strength of composites.

so the tensile strength increases with increasing fiber concentration. It appears that the mechanical bonding between fibers and bitumen binders plays an important role in increasing the tensile strength of bitumen-fiber mastics. Note that the elongation at rupture decreases with increasing fiber contents as shown in Fig. 8. Adding fibers appears to induce brittleness in bitumen-fiber mastics.

Fig. 10 shows that toughness increases with increasing organic fiber concentration up to 0.3%, confirming

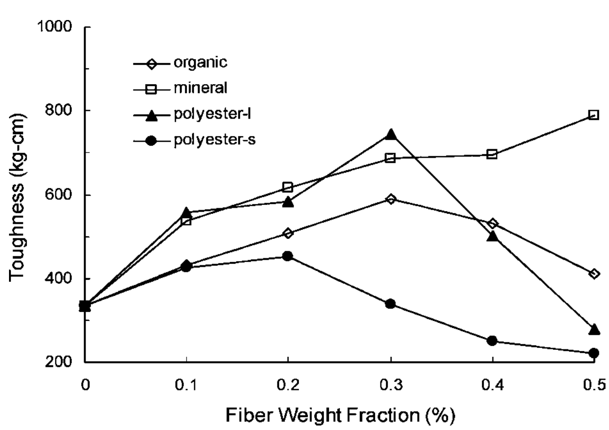


Figure 10 Toughness of bitumen mixing with fibers.

the optimum content of organic fibers decided by the viscosity test. Note that bitumen AC-20's toughness is represented by 0% fiber weight fraction. For organic mastics the toughness value at 0.3% concentration is double than that at 0%. The engineering properties of bitumen are enhanced up to the optimum content of fibers; therefore, the addition of fibers could contribute the performance of asphalt mixtures. The gradual decrease in the toughness after the peak point indicates that adding more than 0.3% organic fibers may result in morphological discontinuity between cellulose and bitumen.

The reduction of toughness after a peak value is also observed in polyester fibers. This type of discontinuity results from fiber-bitumen interaction and fiber–fiber interaction [16, 18]. Due to the increased fiber–fiber interaction and fiber-bitumen incompatibility, toughness decreases with increasing fiber content. This is at least partially the reason why the addition of fibers to bitumen does not necessarily lead to a significant increase in mastic toughness after a critical fiber concentration.

The effect of diameter on toughness is also shown in Fig. 10, which indicates that 6-μm mineral fibers exhibit higher toughness than 90-μm polyester-s fibers. The length of both fibers is 6000 μm; however, the tensile strength of polyester fibers is much higher than that of mineral fibers as listed in Table I. One reason may be that the interfacial area per unit weight of mineral fibers is much higher than that of polyester fibers. At a given fiber concentration, thin fibers have more surface areas to carry tensile loads for bitumen than thick ones, thus increasing the tensile strength of mineral fiber-reinforced bitumen.

Another way to evaluate the effect of fiber diameter on the toughness of mastics is related to the stress distribution around the fiber, as illustrated in Fig. 11. Due to differences in modulus and Poisson's ratio between the fiber and the matrix, stress concentration occurs at the interface. Researchers have shown that the stress concentrations near the fiber are dependent of fiber size under the condition of good adhesion [18, 19].

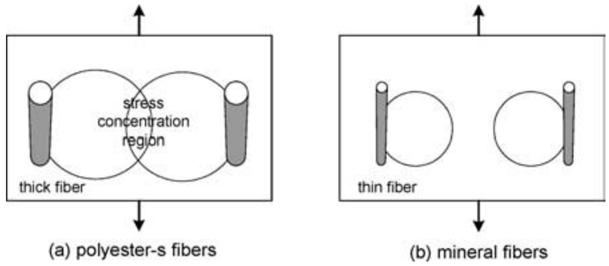


Figure 11 Stress concentrations due to fiber diameter.

The thicker the fiber is, the larger the stress concentration area is. If the distance between fibers is smaller than the stress concentration area, there is an overlap area between fibers, wherein the strength of the fiber-filled composite is reduced.

5.5. Viscoelastic behavior

The complex modulus (G^*) for different concentrations of mineral fibers mixed with bitumen is shown in Fig. 12. For clarity, curves of other blends are not plotted. For pavements to resist rutting, a high G^* value and low tan δ are desired. The higher the G^* value, the stiffer and thus the more resistant to rutting the mastic will be. The lower the tan δ value, the more elastic the mastic. Increasing fiber contents generally leads to an increase in complex modulus. At the concentration level of 0.4%, adding fiber results in a marked increase in G^*, as illustrated in Fig. 12. This 0.4% content of mineral fibers is in good agreement with content determined by the viscosity ratio. Concentrations higher than 0.4% cause a fiber-fiber interaction that could result in the reduction of complex modulus. The decrease in complex modulus is also observed for other fibers when the concentration level is higher than 0.4%.

The rheological parameter, tan δ, is one of the key elements for predicting binder performance [20, 21]. The tan δ trend for organic-reinforced bitumen is shown in Fig. 13. The most significant reduction on tan δ occurs with the addition of 0.3% cellulose fibers, and this observation corresponds well with previous discussion. However, adding more than 0.3% may not be economically feasible because of the limited benefits from reducing the tan δ value.

6. Conclusions

Laboratory tests were performed to quantify the strength mechanism of bitumen mixed with fibers. The following conclusions are drawn based on the test results.

1. The addition of fibers stiffens bitumen, and possesses potential benefits to reduce the amount of

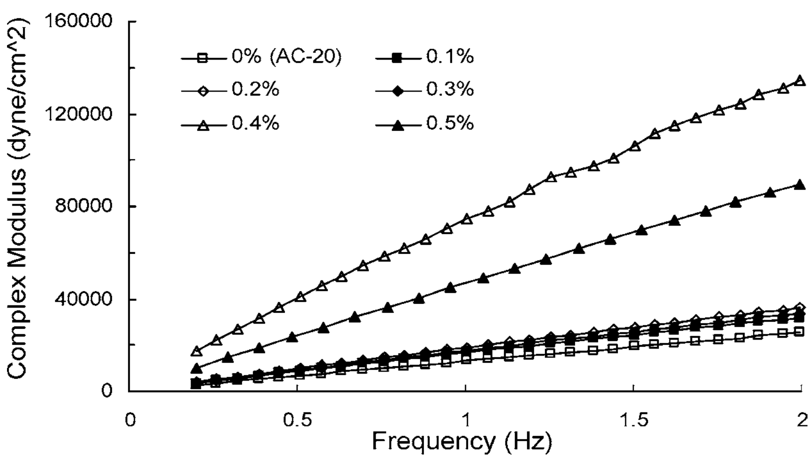


Figure 12 Complex modulus curves at 60°C for mineral fibers mixed with bitumen.

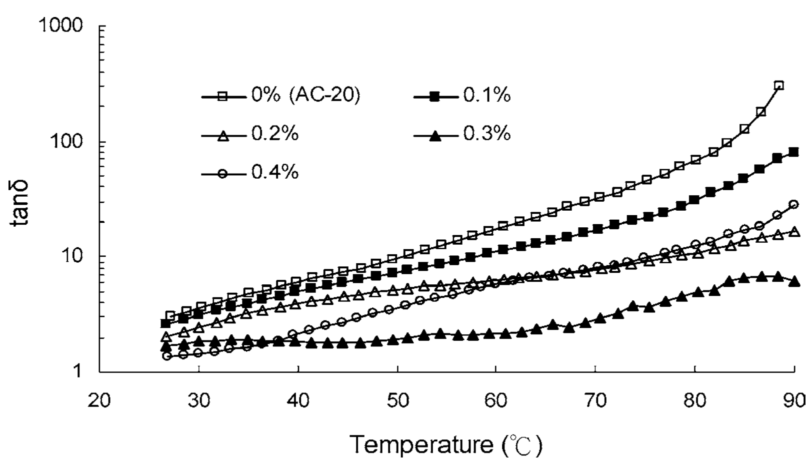


Figure 13 Rheological parameter tan δ versus temperature at 1.59 Hz for bitumen mixing with organic fibers.

draindown. A stiffening effect of ten times higher than bitumen's viscosity appears to be adequate for the strength reinforcement using fibers. In addition, other factors such as cost and pavement performance need to be considered for the fiber selection. Reinforcements more than ten times are costly, and may be detrimental to mixture performance.

2. The stabilizing effect of bitumen mixed with fibers could be explained on the basis of the three-dimensional network in the microspores of the fibers. Good adhesion between fibers and bitumen enhances the load-carrying ability of bitumen-fiber mastics.

3. Cellulose and mineral fibers behave similarly in reinforcing bitumen. Polypropylene fibers tend to be entangled, resulting in higher softening points. If the strength reinforcement of bitumen-fiber mastics is needed at higher temperatures, polyester fibers may be used.

4. The criteria used to select an optimum fiber concentration include viscosity, softening point, toughness and tenacity, and viscoelastic properties. Engineering properties and economic factors are taken into consideration when the proper content of fibers is decided for asphalt mixtures. For fibers tested in this study, the optimum content is found to be 0.3, 0.4, 0.4 and 0.3% by mixture weight for organic, mineral, short polyester and long polyester fibers, respectively.

Acknowledgements
The authors are very grateful to the National Science Council (NSC92-2211-E-006-096) and the Construction and Planning Administration for providing financial support.

References
1. AMERICAN CONCRETE INSTITUTE (ACI), in "Committee 544 Report, ACI 544.1R-82: State-of-the-Art Report on Fiber Reinforced Concrete" (Detroit, 1986) p. 124.
2. T. UMOTO, H. MUTSUYOSHI, F. KATSUKI and S. MISRA, *J. Mater. Civil Engng.* **14** (2002) 191.
3. K. D. STUART and P. MALMQUIST, *Transport. Res. Record.* **1454** (1994) 48.
4. P. V. PELTONEN, *J. Mater. Sci.* **26** (1991) 5618.
5. H. AKBULUT, A. R. WOODSIDE and W. D. H. WOODWARD, Polymer Modified Cellulose Pellet Fiber in Bituminous Mixtures, in Proceedings of 2nd Eurasphalt & Eurobitume Congress, on CD-ROM, Barcelona, April 2000.
6. L. A. COOLEY, E. R. BROWN and D. E. WATSON, in "NCAT 2000-05 Evaluation of OGFC Mixtures Containing Cellulose Fibers" (National Center for Asphalt Technology, Alabama, 2000) p. 87.
7. K. D. STUART, in "FHWA-RD-92-006 Stone Mastic Asphalt (SMA) Mixture Design" (US Department of Transportation, Washington, DC, 1992) p. 156.
8. E. R. BROWN and R. B. MALLIC, *Transport. Res. Rec.* **1513** (1996) 25.
9. Y. DECOENE, *ibid.* **1265** (1990) 82.
10. M. E. DAINES, "Trials of Porous Asphalt and Rolled Asphalt on the A38 at Butron" (Department of Transport TRRL Report RR 323, Crowthorne, UK, 1992) p.125.
11. J. C. NICHOLLS, "Review of UK Porous Asphalt Trials" (Transport Research Laboratory TRL Report 264, Crowthorne, UK, 1997) p. 165.
12. T. M. WILLIAMS and F. P. MIKNIS, *J. Mater. Civil Engng.* **10** (1998) 121.
13. R. L. SCHALEK and L. T. DRZAL, *J. Adv. Mater.* **2** (2000) 32.
14. E. K. BILLS, K. H. SWEENLY and F. S. SALCEDO, *J. Appl. Polym. Sci.* **4** (1960) 259.
15. V. P. CHACKO, F. E. KARASZ and R. J. FARRIS, *Polym. Engng. Sci.* **22** (1982) 968.
16. L. E. NIELSEN, "Mechanical Properties of Polymers and Composite" (Marcel Dekker, Inc., New York, 1974) p. 556.
17. A. A. BERLIN, S. A. VOLFSON, N. S. ENIKOLOPIAN and S. S. NEGMATOV, "Principles of Polymer Composites" (Springer-Verlag, New York, 1986) p. 453.
18. S. Y. FU, B. LAUKE, E. MADER, C. Y. YUE and X. HU, *Composites: Part A.* **31** (2000) 1117.
19. J. N. GOODIER, *J. Appl. Mech.* **55** (1933) 39.
20. C. Y. CHEUNG and D. CEBON, *J. Mater. Civil Engng.* **9** (1997) 117.
21. G. HERNANDEZ, R. RODRIGUEZ, R. BLANCO and V. M. CASTANO, *Intern. J. Polym. Mater.* **35** (1997) 129.
22. J. S. CHEN and C. H. PENG, *J. Mater. Civil Engng.* **10** (1998) 256.
23. G. FRITSCHY and E. PAPIRER, *Rheologica Acta* **28** (1979) 749.

Received 23 January
and accepted 25 August 2004